Electric Potential and Equipotential Surfaces

Objectives

After going through this lesson, the learners will be able to:

- Understand Electric Potential Electric potential difference
- Derive an expression for Electric potential: due to a point charge, a dipole and a system of charges
- Visualize equipotential surfaces
- Calculate electric potential

Content Outline

- Unit syllabus
- Module wise distribution
- Words you must know
- Introduction
- Electrostatic potential and potential difference
- Electric potential due to a point charge,
- Electric potential a system of charges
- Electric potential a uniformly charged thin spherical shell
- Electric potential due to a dipole
- Equipotential surfaces
- Electrostatic potential energy
- Potential energy in an external field of a point charge and a dipole
- Summary

Unit Syllabus

Chapter-1: Electric Charges and Fields

Electric Charges; Conservation of charge, Coulomb's law-force between two point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field; electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.

Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

Chapter-2: Electrostatic Potential and Capacitance

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges; equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.

Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarization, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.

Module Wise Distribution of Unit Syllabus - 11 Modules

The above unit is divided into 11 modules for better understanding.

Module 1	● Electric charge ● Properties of charge ● Coulombs' law ● Characteristics of coulomb force ● Constant of proportionality and the intervening medium ● Examples
Module 2	● Forces between multiple charges ● Principle of superposition ● Continuous distribution of charges ● numerical
Module 3	● Electric field E ● Importance of field E and ways of describing field ● Superposition of electric field ● Examples
Module 4	● Electric dipole ● Electric field of a dipole ● Charges in external field ● Dipole in external field Uniform and non-uniform

Module 5	• Electric flux, • Flux density • Gauss theorem • Application of gauss theorem to find electric field • For a distribution of charges • Numerical
Module 6	• Application of gauss theorem Field due to field infinitely long straight wire • Uniformly charged infinite plane • Uniformly charged thin spherical shell (field inside and outside) • Graphs
Module 7	• Electric potential, • Potential difference, • Electric potential due to a point charge, a dipole and system of charges; • Equipotential surfaces, • Electrical potential energy of a system of point charges and of electric dipole in an electrostatic field. • Numerical
Module 8	• Conductors and insulators, • Free charges and bound charges inside a conductor. • Dielectrics and electric polarization
Module 9	• Capacitors and Capacitance, • Combination of capacitors in series and in parallel • Redistribution of charges, common potential • numerical
Module 10	• Capacitance of a parallel plate capacitor with and without dielectric medium between the plates • Energy stored in a capacitor
Module 11	• Typical problems on capacitors

Module 7

Words You Must Know

Let us recollect the words we have been using in our study of this physics course.

- **Electric Charge:** Electric charge is an intrinsic characteristic of many of the fundamental particles of matter that gives rise to all electric and magnetic forces and interactions. There are two kinds of charges, positive and negative.
- **Conductors:** Some substances readily allow passage of electricity through them, others do not. Those which allow electricity to pass through them easily are called *conductors*. They have electric charges (electrons) that are comparatively free to move inside the material. Metals, humans, animal bodies and earth are all conductors of electricity.
- **Insulators:** Most of the non-metals, like glass, porcelain, plastic, nylon, wood, offer high opposition to the passage of electricity through them. They are called *insulators.*
- **Point Charge:** When the linear size of charged bodies is much smaller than the distance separating them, the size may be ignored and the charge bodies can then be treated as *point charges*.
- **Conduction:** Transfer of electrons from one body to another, it also refers to flow of charged electrons in metals and ions in electrolytes and gases.
- **Induction:** The temporary separation of charges in a body due to a charged body in the vicinity. The effect lasts as long as the charged body is held close to the body in which induction is taking place.
- **Quantization of charges:** Charge exists as an integral multiple of basic electronic charge. Charge on an electron is 1.6×10^{-19} C.
- **Electroscope:** A device to detect charge, to find the relative magnitude of charge on two charged bodies. A suitably charged electroscope can be used to find the nature of charge on a charged body.
- **Coulomb:** S.I unit of charge defined in terms of 1 ampere current flowing in a wire to be due to 1 coulomb of charge flowing in 1 s.

  1 coulomb = collective charge of 6×10^{18} electrons
- **Conservation of charge:** Charge can neither be created nor destroyed in an isolated system; it (electrons) only transfers from one body to another.
- **Coulomb's Force:** It is the electrostatic force of interaction between the two point charges.

- **Coulomb's law:** A mathematical expression based on coulomb's law to show the magnitude as well as direction of mutual electrostatic force between two or more charges.

$$F = K\frac{q_1 \times q_2}{r^2}$$

- **Vector form of Coulomb's law:** A mathematical expression based on coulomb's law to show the magnitude as well as direction of mutual electrostatic force between two or more charges. Force between charges q_1 and q_2, F_{12} is the force on 1 due to 2, depending upon the nature of the charges (both positive, both negative or q_1 positive and q_2 negative or q_2 positive and q_1 negative)

$\hat{r}_{12}$ vector shows the direction of the force

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

Laws of vector addition:

- **Triangle law** of vector addition: If two vectors are represented by two sides of a triangle in order, then the third side represents the resultant of the two vectors.
- **Parallelogram law** of vector addition: If two vectors are represented in magnitude and direction by adjacent sides of a parallelogram then the resultant of the vectors is given by the diagonal passing through their common point.

Also resultant of vectors P and Q acting at angle of θ is given by

$$R = \sqrt{P^2 + Q^2 + 2PQcos\theta}$$

- **Polygon law** of vector addition: Multiples vectors may be added by placing them in order of a multi sided polygon, the resultant is given by the closing side taken in opposite order.
- **Linear charge density:** The *linear charge density, λ* is defined as the charge per unit length.
- **Surface charge density:** The surface *charge density* σ is defined as the charge per unit surface area. The surface charge density σ at the area element Δs is given by σ = $\frac{\Delta Q}{\Delta s}$.
- **Volume charge density:** The volume ch*arge density* ρ is defined as the charge per unit volume.

- **Superposition Principle:** For an assembly of charges q_1, q_2, q_3, ..., the force on any charge, say q_1, is the vector sum of the force on q_1 due to q_2, the force on q_1 due to q_3, and so on. For each pair, the force is given by Coulomb's law for two point charges.
- **Electric field lines:** An electric field line is a curve drawn in such a way that the tangent at each point on the curve gives the direction of the electric field at that point.
- **Area vector:** The area element vector ΔS at a point on a closed surface equals $\Delta S\,\hat{n}$ where ΔS is the magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal at that point.
- **Gaussian surface:** The closed surface that we need to choose for applying Gauss's law to a particular charge distribution is called the Gaussian surface.
- **Gauss's Theorem/Law:** The flux of the electric field through any closed surface S is $1/\varepsilon_0$ times the total charge enclosed by that surface.
- **Electric field** the space around a charge where its influence may be experienced by other charged bodies. The field strength at appoint in the field is given by E =electrostatic force per unit charge; unit is NC^{-1}
- **Electric field line** electric field lines in an electric field which trace the path of a unit positive charge
- **Electric flux** electric field lines crossing an area
- **Electric flux density** field lines crossing a unit area held perpendicular to the field lines represented by φ unit weber

  $\phi = E.\Delta s$

Introduction

When an external force does work in taking a body from a point to another against a force like spring force or gravitational force; that work gets stored as potential energy of the body. When the external force is removed, the body moves, gaining kinetic energy and losing an equal amount of potential energy. The sum of kinetic and potential energies is thus conserved. Forces of this kind are called conservative forces. **Spring force and gravitational force are examples of conservative forces.**

The Coulomb force between two (stationary) charges is also a conservative force.

This is not surprising, since both have inverse-square dependence on distance and differ mainly in the proportionality constants – the masses in the gravitational law are replaced by charges in Coulomb's law.

Thus, like the potential energy of a mass in a gravitational field, we can define electrostatic potential energy of a charge in an electrostatic field.

Electrostatic Potential and Potential difference

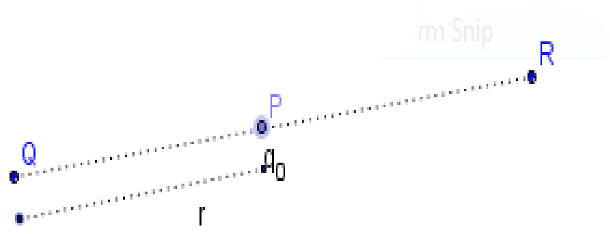


Fig shows: A test charge (q > 0, means a positive charge) is moved from the point R to the point P against the repulsive force on it by the charge Q (> 0) placed at the origin.

Consider an electrostatic field $\vec{E}$ due to some charge Q placed at the origin. Now, imagine that we bring a test charge q from a point R to a point P against the repulsive force on it due to the charge Q.

This will happen if Q and q are both positive and both negative.

For definiteness, let us take Q, q > 0.

Two assumptions may be made here:

First,

We assume that the test charge q is so small that it does not disturb the original configuration, namely the charge Q at the origin.

Second,

In bringing the charge q from R to P, we apply an external force, $\vec{F}_{ext}$ just enough to counter the repulsive electric force $\vec{F}_E$ (i.e., $\vec{F}_{ext} = -\vec{F}_E$). This means there is no net force on or acceleration of the charge q when it is brought from R to P, i.e., it is brought with infinitesimally slow constant speed.

Thus, work done by external forces in moving a charge q from R to P is:

$$W_{R\to P} = \int_R^P \vec{F}_{ext}.\vec{dr}$$

$$W_{R\to P} = -\int_R^P \vec{F}_E.\vec{dr}$$

This work done is against electrostatic repulsive force and gets stored as the potential energy of the charge.

$$\Delta U = U_P - U_R = W_{R\to P}$$

$$\mathbf{V(r) = V_P} = \frac{W_{\infty\to P}}{q_0} = \frac{U_P}{q_0} = \frac{q_0 Q}{4\pi\varepsilon_0 r/q_0}$$

$$V_{(r)} = V_P = \frac{Q}{4\pi\varepsilon_0 r}$$

The S.I unit of electric potential is J/C or volt.

Potential difference between any two point P & R is given by:

$$V_P - V_R = W_{R\to P}/q_0 = U_P - U_R/q_0$$

$$W_{R\to P} = q_0 [V_P - V_R]$$

Note: here that this displacement is in an opposite sense to the electric force and hence work done by electric field is negative, i.e. – $W_{R\to P}$**)**

Therefore, we can define **electric potential energy difference between two points as the work required to be done by an external force in moving (without accelerating) charge *q* from one point to another for an electric field of any arbitrary charge configuration.**

Potential energy of a point charge 'q' may be defined as the amount of work done in bringing a charge 'q' from infinity to that point against the force of repulsion due to charge 'Q' without any acceleration.

Similar deduction for potential is true for any sign of charge although we have derived it for $Q > 0$, for $Q < 0$ $V < 0$

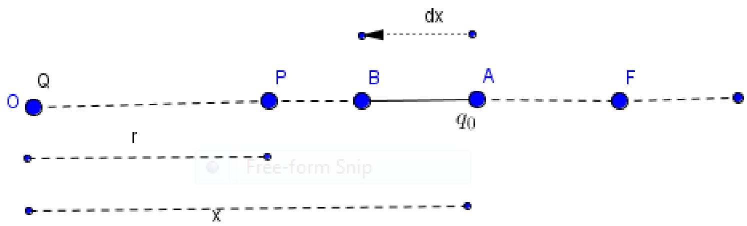
Work done by the external force per unit positive test in bringing it from infinity to the point is negative. This is equivalent to saying that the work done by the electrostatic force in bringing the unit positive charge from infinity to the point P is positive (this is as it should be if Q < 0 the force on a unit positive test charge is attractive, so that the electrostatic force and displacement from infinity to our point P are in the same direction.

This helps us to describe infinity. In the context, the potential at infinity if zero or it is the boundary of the electrostatic field (the region of influence) of the charge, or charges in consideration.

Electric Potential due to a Point Charge

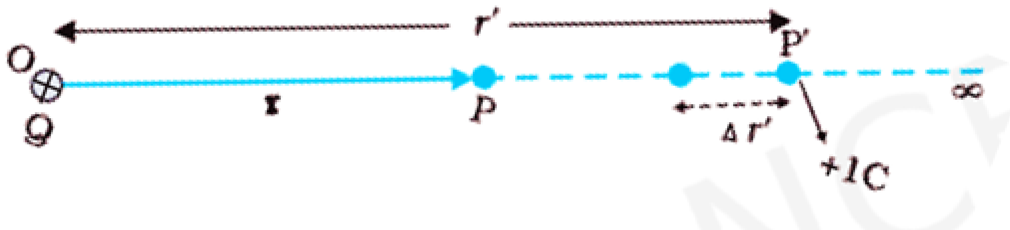
Electrostatic potential 'V' at a point 'P' in an electric field of a point charge Q is equal to the amount of work done in bringing unit test charge from infinity to that point (P) against the repulsion of the field without any acceleration.

Consider a point charge Q at the origin as shown in the Fig.



For definiteness, we take Q to be positive. We wish to determine the potential at any point P with position vector r from the origin. For that we must calculate the work done in bringing a unit positive test charge from infinity to the point P.

For $Q > 0$, the work done against the repulsive force on the test charge is positive. Since work done is independent of the path, we choose a convenient path – along the radial direction from infinity to the point P.



Our point charge Q is placed at origin O.

Electric potential at point P will be equal to the amount of work done in bringing a unit positive test charge from infinity to the point P.

The force $\vec{F}$ acts away from the charge Q. the small work done in moving the test charge q_0 from A to B through small displacement 'dx' against the electrostatic force is :

$$dW = \vec{F}.\vec{dr} = F\, dr \cos 180° = -F.dr$$

Think Why 180^0?

Total work done (W) by the external force is obtained by integrating

The negative sign appears because for $\Delta r' < 0, W$ is positive.

$$W = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r'^2} dr' = \left| \frac{Q}{4\pi\varepsilon_0 r'} \right|_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

Using rules of integration

$$W_{\infty\rightarrow P} = -\int_{\infty}^{r} F dr = -\int_{\infty}^{r} q_0 E dr = -q_0 \int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0} r^2 dr$$

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

Thus the electric potential due to a point charge is spherically symmetric and it depends only on the distance of the observation point from the charge and not on the directions from the charge

This means at all locations at distance x from the charge in 3 dimensional space, we have the same potential.

Here, we have assumed that the potential energy of the system is zero at infinity.

Potential energy difference (ΔU) depends only on initial and final positions and is independent of path followed in going from one point to other because the electrostatic force is conservative

Example:

(a) Calculate the potential at a point P due to a charge of 4×10^{-7}C located 9 cm away.

(b) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Solution:

(a) $V(r) = \frac{Q}{4\pi\varepsilon_0 r} = 9\times10^{9}\, Nm^{2} C^{2} \times \frac{4\times10^{-7} C}{0.09\, m} = 4\times10^{4} V$

(b) $W = qV = 2\times10^{-9} C \times 4\times10^{4} V = 8\times10^{-5} J$

No, work done will be path independent. Any arbitrary infinitesimal path can be resolved into two perpendicular displacements: One along r and another perpendicular to r. The work done corresponding to the later will be zero.

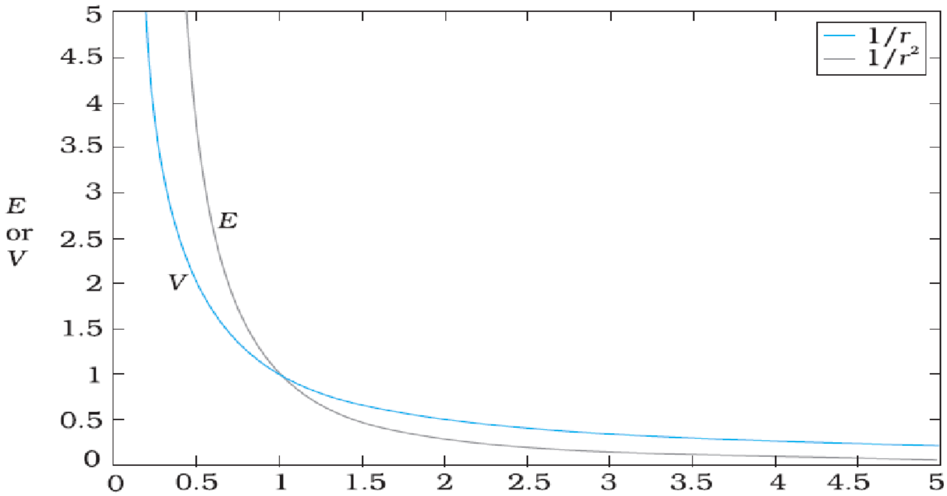
Example:

Show the Variation of potential V with r [in units of $(Q/4\pi\varepsilon_0)$ m^{-1}] and electric field E with r in units of $(Q/4\pi\varepsilon_0)$ m^{-2}] for a point charge Q.

Solution:

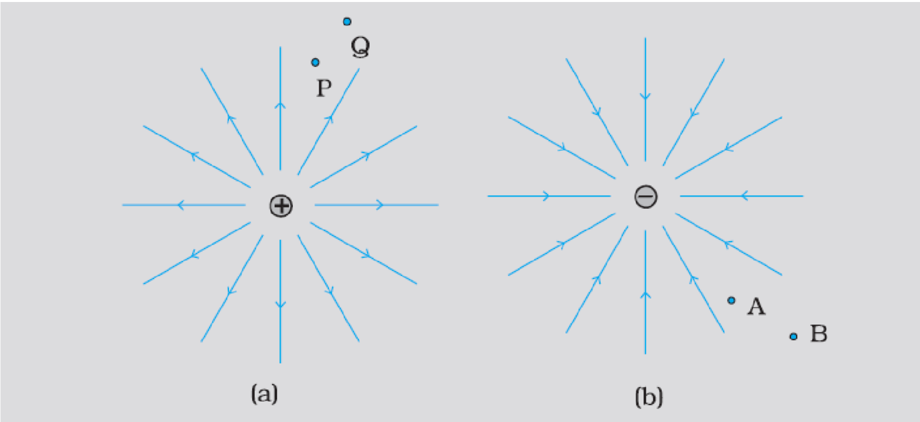
The blue curve is showing variation of potential with distance for a point charge

The black curve shows the variation of electric field with distance for a point charge



Example:

Figures (a) and (b) show the field lines of a positive and negative point charge respectively



(a) Give the signs of the potential difference $V_p - V_q$; $V_B - V_A$.

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

(d) Give the sign of the work done by the external agency in moving a small negative charge from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

Solution:

(a) As $V \propto \frac{1}{r}$; $V_P > V_Q$. $Thus,\ \left(V_P - V_Q\right) is\ positive.$

Also $V_B\ is\ less\ negative\ than\ V_A. Thus,\ V_B > V_A or\ \left(V_B - V_A\right) is\ positive.$

(b) A small negative charge will be attracted towards positive charge. The negative charge moves from higher potential energy to lower potential energy. Therefore, the sign of potential energy difference of a small negative charge between Q and P is positive.

Similarly, (P.E.) A > (P.E.) B and hence the sign of potential energy differences is positive.

(c) In moving a small positive charge from Q to P, work has to be done by an external agency against the electric field. Therefore, work done by the field is negative.

(d) In moving a small negative charge from B to A work has to be done by the external agency. It is positive.

(e) Due to the force of repulsion on the negative charge, velocity decreases and hence the kinetic energy decreases in going from B to A.

Example:

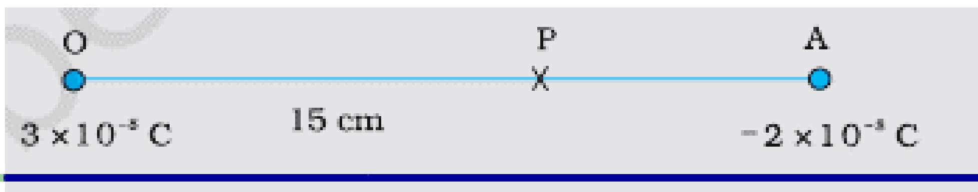
Two charges 3×10^{-8} C and -2×10^{-8}C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero?

Take the potential at infinity to be zero.

Solution:

Let us take the origin O at the location of the positive charge.

The line joining the two charges is taken to be the *x*-axis; the negative charge is taken to be on the right side of the origin.



Let P be the required point on the x-axis where the potential is zero.

If x is the x- coordinate of P, obviously x must be positive.

(There is no possibility of potentials due to the two charge)

adding up to zero for $x < 0$.) If x lies between O and A, we have

$$\frac{1}{4\pi\epsilon_0}\left[\frac{3\times10^{-8}}{x\times10^{-2}} - \frac{2\times10^{-8}}{(15-x)\times10^{-2}}\right] = 0$$

Where x is in cm, hence

$$\frac{3}{x} - \frac{2}{15-x} = 0$$

Which gives x=9 cm

If x lies on the extended line OA, the required condition is

$$\frac{3}{x} - \frac{2}{x-15} = 0$$

x = 45 cm

Thus, electric potential is zero at 9 cm and 45 cm away from the positive charge on the side of the negative charge. Note that the formula for potential used in the calculation required choosing potential to be zero at infinity.

Example:

A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Solution:

Work done = potential difference between P and Q or work done in carrying the charge of -2×10^{-9} C from P to Q

$$W_{PQ} = q\left(V_P - V_Q\right)$$

$$W_{PQ} = 8\times10^{-3}C\times9\times10^{9} \times \left(-\ 2\times10^{-9}\right)\left[\frac{1}{3\times10^{-2}} - \frac{1}{4\times10^{-2}}\right]V$$

$$=-\ 144\times10^{-3} \times 10^{2}\left(-\frac{1}{12}\right) = 1.2J$$

Even if the path is made different the work done remains the same as electrostatic force is a conservative force.

Electric Potential due to a System of charges

Consider a system of charges q_1, q_2, ...q_n with position vectors r_1, r_2, ..., r_n relative to some origin.

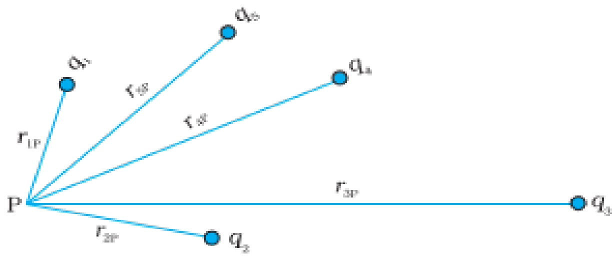


Fig: Potential at a point due to a system of charges is the sum of potentials due to individual charges.

The potential V_1 at P due to the charge q_1 is:

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$$

Where, r_{1P} is the distance between q_1 and P

Similarly, the potential at point P due to other charges will be given by:

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}}, \quad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}}$$

Therefore, **total potential V at point P due to all charges is obtained by superposition principle which is equal to algebraic sum of potential due to the individual charges at that point.**

$$V = V_1 + V_2 + ... + V_n$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + ... \frac{q_n}{r_{nP}}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}}$$

Why not vector sum?

Example:

Two charges $4 \times 10^{-8} C$ and $-3 \times 10^{-8} C$ are located 12 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Solution:

Let us take the origin O at the location of the positive charge. The line joining the two charges is taken to be the *x*-axis; the negative charge is taken to be on the right side of the origin.

Let P be the required point on the *x*-axis where the potential is zero. If *x* is the *x*-coordinate of P, obviously *x* must be positive. (There is no possibility of potentials due to the two charges adding up to zero for $x < 0$.) If *x* lies between O and A, we have:

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{4\times10^{-8}}{x\times10^{-2}} - \frac{3\times10^{-8}}{(12-x)\times10^{-2}}\right] = 0$$

$$\frac{4\times10^{-8}}{x\times10^{-2}} - \frac{3\times10^{-8}}{(12-x)\times10^{-2}} = 0$$

$$\frac{4}{x} - \frac{3}{(12-x)} = 0$$

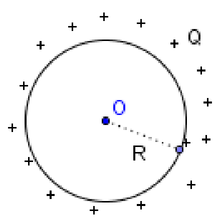
This gives:

$x = 6.85cm$

Electric Potential due to a Uniformly Charged thin Spherical Shell

Consider a uniformly charged spherical shell of radius R and carrying charged Q.

To calculate potential at point P at a distance r from its center o is as shown:



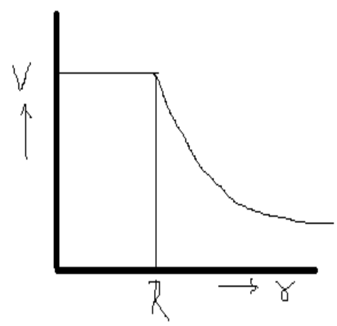
For a uniformly charged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the centre. Thus, the potential outside the shell is given by

$$V = \frac{q}{4\pi\varepsilon_0 r} \qquad (r \geq R)$$

Where q is the total charge on the shell and R is its radius.

The electric field inside the shell is zero.

This implies that potential is constant inside the shell (as no work is done in moving a charge inside the shell), and, therefore, equals its value at the surface, which is



$V = \frac{q}{4\pi\varepsilon_0 R}$

Graph showing the variation of potential from the center of a charged spherical hollow shell.

Example:

A spherical conductor of radius 12 cm has a charged 1.6 $\times 10^{-7}$ C distributed uniformly on its surface. What is electric potential?

(a) inside the sphere

(b) just outside the sphere

(c) At a point 18 cm from the center of the sphere?

Solution:

$$V = kQ/R \quad for\, r \leq R$$

$$V = kQ/r \quad for\, r > R$$

(a) $V\,(inside) = \dfrac{kQ/R = 9\times10^{9}\times1.6\times10^{-7}}{12\times10^{-2}} = 1.2 \times 10^{-2}\, volt$

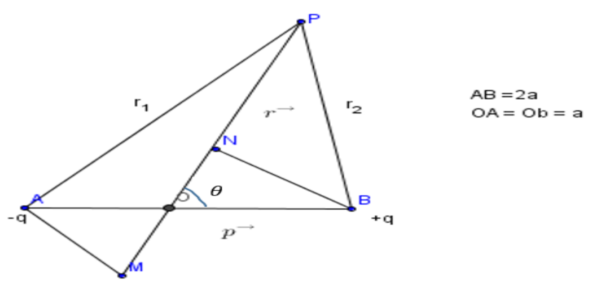
(b) $V\,(at\, surface) = kQ/R = 1.2 \times 10^{4}\, V$

(c) For potential at any point outside the shell:

$$V = kQ/r = \frac{9\times10^{9}\times1.6\times10^{-7}}{18\times10^{-2}} = 0.8 \times 10^{4} V = 8 \times 10^{3}\, V = 8\, kV$$

Potential due to an Electric Dipole

Electric dipole is a system of two equal and opposite charges (-q) & (+q) separated by (small) distance (2a). Its total charge is zero. It is characterized by a dipole moment $\vec{p}$ whose magnitude is: ($p = q\times2a$) and pointed in direction (–q to +q).



Net potential at any point P at a distance r from the midpoint of dipole in a direction OP making angle θ with dipole moment p is given by using superposition principle:

$$V_P = potential\ at\ P\ due\ to - q + potential\ at\ P\ due\ to + q = V_1 + V_2$$

$$VP = +\frac{kq}{r_1} + \frac{k(-q)}{r_2} = k\,q\,[1/r_1 - 1/r_2]$$

$$\therefore r1 = AP \approx MP = OP + OM = r + a\cos\theta$$

$$r2 = BP \approx NP = OP - ON = r - a\cos\theta$$

$$\therefore V = kq\left[\frac{1}{r-a\cos\theta} - \frac{1}{r+a\cos\theta}\right]$$

$$= kq\left[\frac{r+a\cos\theta - r + a\cos\theta}{r^2-(a\cos\theta)^2}\right]$$

$$= \frac{kq\times 2a\times\cos\theta}{\left(r^2-(a\cos\theta)^2\right)}$$

$$= \frac{kp\cos\theta}{r^2-(a\cos\theta)^2}$$

$$where\ p = q\times 2a$$

$$= \frac{k\vec{p}.\hat{r}}{r^2-(a\cos\theta)^2}$$

$$V \approx k\frac{\vec{p}.\hat{r}}{r^2} \quad (where,\ r >>> a)$$

Special Cases:

A. **When the point 'P' lies on the axial line of the dipole ,θ= 0^0 or 180^0**; and $V = \pm kp/r^2$

B. When the point 'P' lies on the equatorial line of the dipole, θ = **90^0 and V = 0**. However, the electric field at such points is non-zero.

The important contrasting features of electric potential of a dipole and electric potential due to a point charge are:

- **The potential due to a dipole depends not just on r but also on the angle between the position vector $\vec{r}$ and the dipole moment $\vec{p}$ it is however symmetric about p .which means, if we rotate the position vector r about p keeping θ fixed .The points corresponding to P on the cone so generated will have the same potential as at P.**
- **The electric dipole potential falls off, at large distances, as $1/r^2$, and not as $1/r^2$ for a single charge.**

Example:

A short electric dipole has a dipole moment of 4×10^{-9} C cm. Determine the electric potential due to the dipole at a point distance 0.3m from the center of dipole situated:

(a) on the axial line

(b) on the equatorial line

(c) on a line making an angle of 60^0 with the dipole axis.

Solution:

$$p = 4\times 10^{-9} cm\ and\ r = 0.3cm$$

(a) potential at a point on axial line is:

$$V = kp\cos 0° / r^2 = 9\times10^9 \times 4\times10^{-9} / (0.3)^2 = 400\ V$$

(b) potential at a point on equatorial line is:

$$V = \frac{kp\cos 90°}{r^2} = 0$$

(c) potential at a point on line making $\theta = 60^0$ with $\vec{p}$ is:

$$V = \frac{kp\cos 60°}{r^2} = \frac{9\times10^9 \times 4 \times 10^{-9} \times 1/2}{(0.3)^2} = 200V$$

Example:

Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$, respectively.

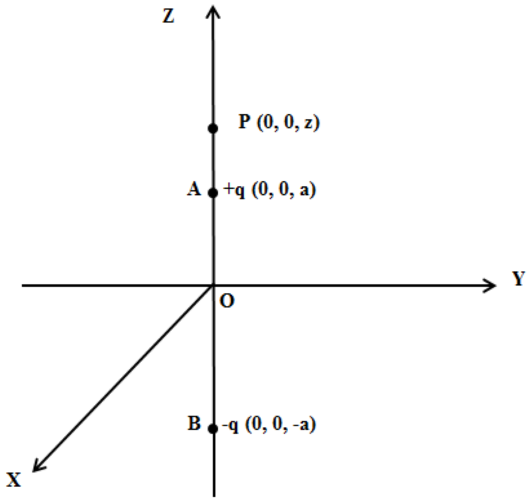
What is the electrostatic potential at the points $(0, 0, \pm z)$ and $(x, y, 0)$?

Solution:

Let us read the question once again carefully and note down the given parameters

Using symbols, we can write

The charge (-q) is located on the negative side of the z-axis at a distance 'a' from the origin 'O' and the charge +q is located on the positive side of the z-axis at a distance of 'a' from the origin.



To find the potential at the point p (0, 0, z), let us find the distance of this point from the given charges.

Distance from the charge +q = AP = r_1 = z - a

Distance from the charge - q = BP = r_2 = z + a

Potential at P due to charge +q = $V_1 = k\frac{q}{r_1} = \frac{1}{4\pi\epsilon_0}\frac{q}{(z-a)}$

Potential at P due to charge - q = $V_2 = -\ k\frac{q}{r_2} = -\frac{1}{4\pi\epsilon_0}\frac{q}{(z+a)}$

Net Potential at P, $V = V_1 + V_2 = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{z-a} - \frac{1}{z+a}\right)$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{z+a-(z-a)}{z^2-a^2}\right)$$

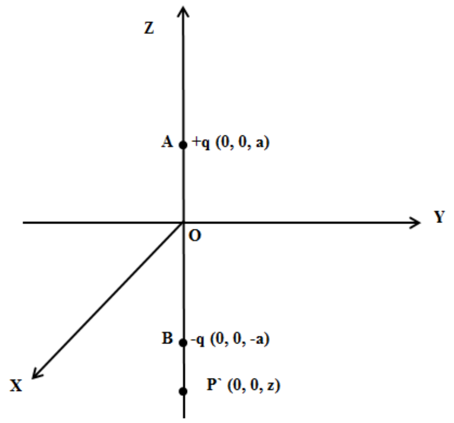
$$= \frac{q}{4\pi\epsilon_0}\left(\frac{2a}{z^2-a^2}\right)$$

Similarly, when we want to find the potential at the point P`(0,0,-z)

Then the distance of the point from +q charge = AP`

= r_1 = z + a

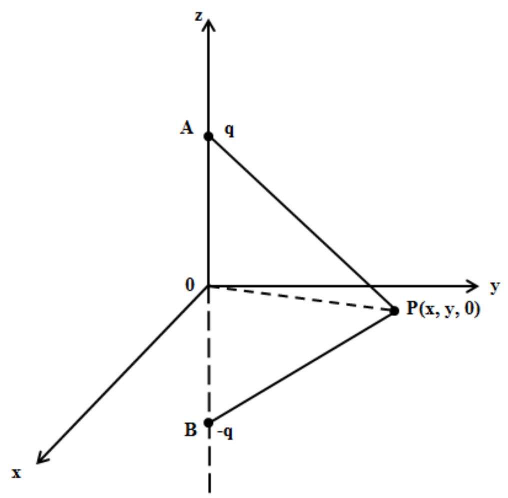
The distance of the point from –q charge = BP` = r_2



$$\mathbf{V_1} = k\frac{q}{z+a} - k\frac{q}{z-a} = kq\left(\frac{z-a-z-a)}{z^2-a^2}\right)$$

$$= kq\left(\frac{-2a}{z^2-a^2}\right)$$

Part II



The point P (x, y, z) lies in the X-Y plane, which is the perpendicular bisector of the z-axis.

This point p will be at equal distance from the charges -q and +q i.e. AP = BP = r

Potential at P due to +q charge and due to –q charge will be equal but negative of each other.

$V_{net} = 0$

Note: In the same way, solve the Q. No. 2.21 of NCERT.

Two charges –*q* and +*q* are located at points (0, 0, –*a*) and (0, 0, *a*), respectively.

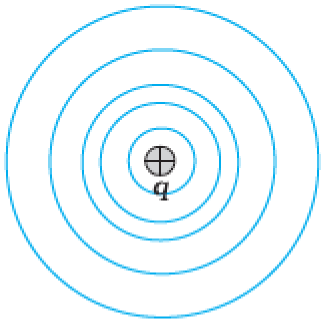
- **What is the electrostatic potential at the points (0, 0, *z*) and (*x*, *y*, 0)?**
- **Obtain the dependence of potential on the distance *r* of a point from the origin when *r*/*a* >> 1.**
- **How much work is done in moving a small test charge from the point (5,0,0) to (–7,0,0) along the *x*-axis?**
- **Does the answer change if the path of the test charge between the same points is not along the *x*-axis?**

Equipotential Surfaces

Any surface which has the same electric potential at every point on it is known as equipotential surface. For a single charge Q, the potential at any point is given by:

$V(r) = \frac{q}{4\pi\varepsilon_0 R}$

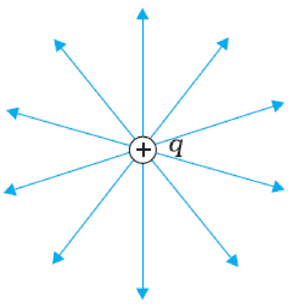
For a single charge *q*



a. Equipotential surfaces are spherical surfaces centered at the charge, and

This would be the case even for a negative charge placed in vacuum.

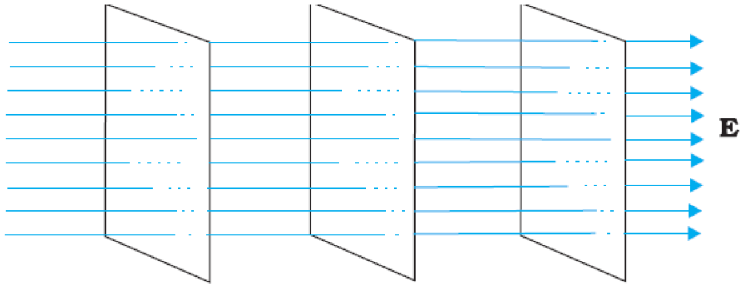
For the same charge we can visualize the electric field lines



(b)

b. Electric field lines are radial, starting from the charge if $q > 0$

For any charge configuration, equipotential surface through a point is normal to the electric field lines at that point.



Equipotential surfaces for a uniform electric field.

If the field lines were not normal to the equipotential surface, it would have non-zero components along the surface.

To move a unit test charge against the direction of the component of the field, work would have to be done. But this is in contradiction to the definition of an equipotential surface: there is no potential difference between any two points on the surface and no work is required to move a test charge on the surface.

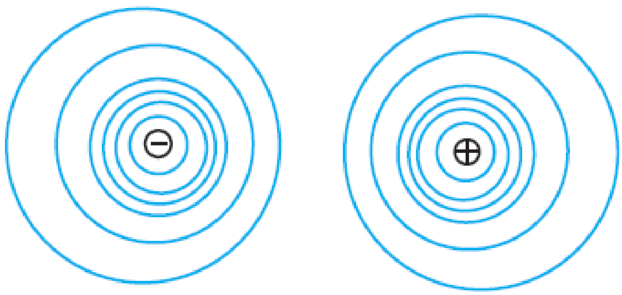
The electric field must, therefore, be normal to the equipotential surface at every point.

Equipotential surfaces offer an alternative visual picture in addition to the picture of electric field lines around a charge configuration.

For a uniform electric field E, say, along the x -axis, the equipotential surfaces are planes normal to the x -axis, i.e., planes parallel to the y-z plane.

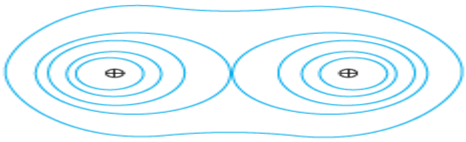
Equipotential surfaces for

a. **A dipole**



(a)

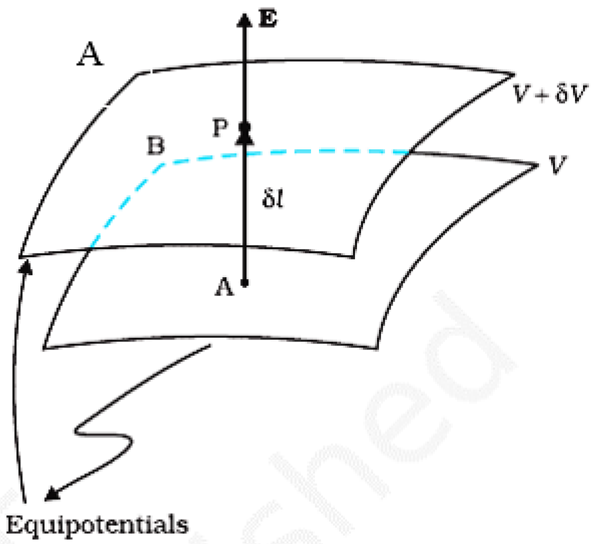
b. **Two identical positive charges**



Relation between Electric Field and Potential

Let us consider two equipotential surfaces A and B with potentials V and V+ dV, where dV is change in potential in the direction of electric field E.

Let P be a point on the surface B. dx is perpendicular distance from the surface A to P. Imagine a test charge q_0 is moved along this perpendicular from surface B to surface A against the electric field.



The work done in the process is:

$$dW_{B\to A} = q_0 [V_A - V_B]$$

$$= q_0 [V - (V- dV)]$$

$$= q_0 dV$$

Same we can calculate as:

$$dW_{B\to A} = \vec{F}.\vec{ds} = -q_0 E \, dx$$

∴ Adding work done from both the process

$dV = - Edx$

∴ $E = - dV/dx$

Or

$$V = -\left|\vec{E}\right| = +\frac{|dV|}{dx}$$

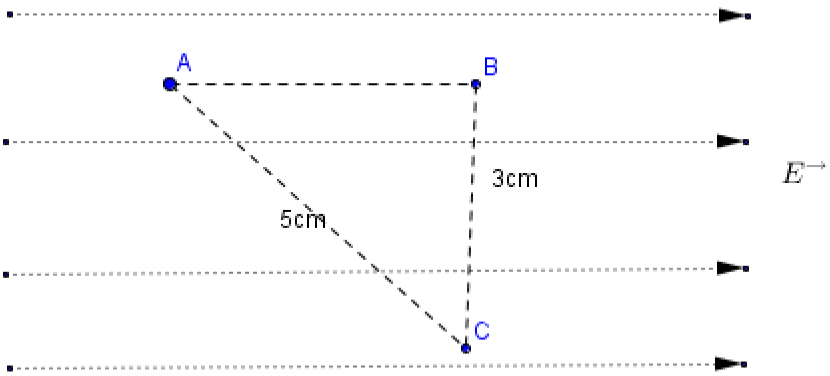
Two important conclusions can be drawn from the above relation between electric field and potential:

(1) Electric field is in the direction in which potential decreases steepest.

(2) Its magnitude is given by the change in magnitude of potential per unit displacement normal to the equipotential surface at that point.

Example:

Three points A, B and C lie in a uniform electric field E of 5 x10^3 N/C as shown in figure. Find the potential difference between A and C.



Solution:

Point B and C lies on same equipotential surface,

$$\therefore V_B = V_C$$

$$\therefore V_A - V_C = (V_A - V_B) + (V_B - V_C)$$

$$= + E\,dx + 0$$

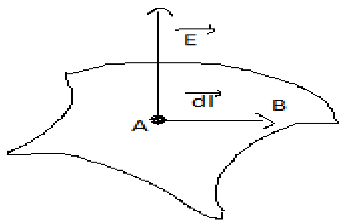
Where, $(dx = AB = \sqrt{AC^2 - BC^2}) = \sqrt{5^2 - 3^2} = 4cm)$

$$V_A - V_C = 5\times10^3\times4\times10^{-2}$$

$$= 200 \text{ V}$$

Properties of equipotential surfaces

- **No work is done in moving a charge over an equipotential surface**:



$W_{B\to A} = q\,[V_B - V_A]$

$= q \times 0$

$= 0$ J

$\therefore$ $V_A = V_B$ at equipotential surface.

- **Electric field is always normal to the equipotential surface at every point**.

For any two points A and B on Equipotential surface:

$W_{A\to B} = q\,[V_B - V_A]$

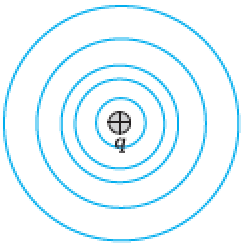
$= q\int dV \int_A^A \vec{E}.\vec{dl}$

$= q \times (-1)$

$= 0$

Which can only happen when $\vec{E}$ is perpendicular to the equipotential surface.

- **Equipotential surfaces are closer together in the regions of strong field and further apart in the regions of weak field.**



$E = - dV / dr$

$dr = \frac{-dV}{E}$

$\therefore$ **For given potential difference** dV **= constant** $dr \propto \frac{1}{E}$

Hence, the gap between the equipotential surfaces will be smaller in the regions, where the electric field is stronger and vice-versa.

- **No two equipotential surfaces can intersect each other**. If they intersect then there will be two values of electric potential at the point of intersection, which is impossible.

Electrostatic Potential Energy:

It is energy possessed by a system of charges by virtue of their positions when two charges are at infinite distance apart, their potential energy is zero because no work has to be done in moving one charge at infinite distance from the other. But when they are brought closer to one another, work has to be done against the force of repulsion.

This work done gets stored as the potential energy.

Potential energy of a system of two point charges:

Suppose a point charge q_1, shown in fig. is at rest at a point A in space. It will produce an electric field around it and hence work has to be done in bringing q_2 from infinity to point B in the field of q_1.

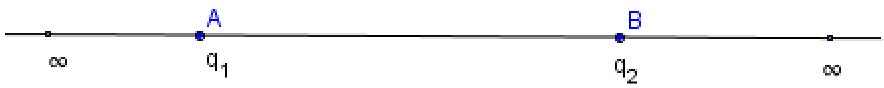


fig. P.E. of two point charges

W_1 = work done in bringing q_1 from ∞ to B when q_1 is at ∞=0

W_2 = work done in bringing unit charge from ∞ to point B

= $q_2 \times$ potential at B due to charge q_1

= $q_2 \times kq_1/r_{12}$

$W_2 = kq_1q_2/r_{12}$ (where r_{12} = distance between points A and B).

As the work done in collecting charges q_1 & q_2 from ∞ to their respective positions at A & B respectively are stored as the potential energy U of the system,

$$\therefore\ U = W_1 + W_2 = 0 + k\frac{q_1q_2}{r_{12}}$$

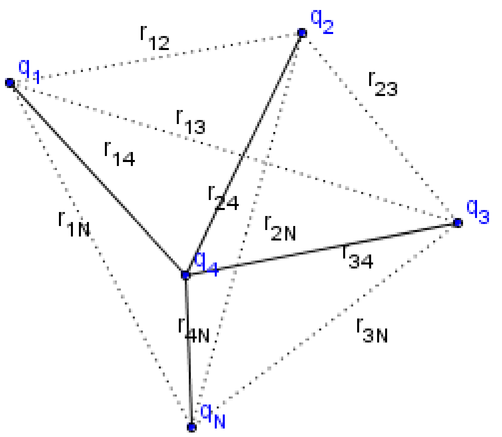
i.e. $U = k\frac{q_1q_2}{r_{12}}$

$\therefore$ U >0; when $q_1q_2 > 0$

& U > 0; when $q_1q_2 < 0$

Potential energy of a system of N point charges:

If $q_1, q_2, q_3, q_4, ...q_n$ are placed in a space as shown in fig;



Then the potential energy of the system is equal to the sum of work done in collecting them from infinity to their respective positions.

$$U = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} k\frac{q_i q_j}{r_{ij}} \qquad \text{where, } i \neq j$$

As double summation counts every pair twice, to avoid this factor ½ has been introduced.

For three charges q_1, q_2, q_3 system:

Potential energy $\qquad U = k\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$

And for four charge system it will be as:

Potential energy

$$U = k\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right]$$

Example:

a. Determine the electrostatic potential energy of a system consisting of two charges $7\ \mu C$ and $-2\ \mu C$ (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively.
b. How much work is required to separate the two charges infinitely away from each other? Also called dissociation energy.

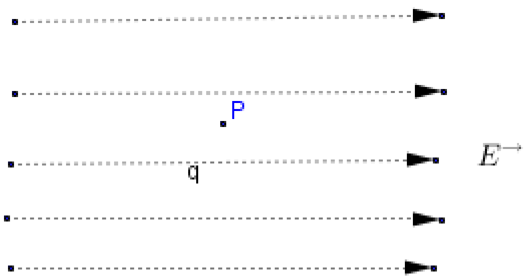
Solution:

(a) $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = 9\times10^9 \times \frac{7\times(-2)\times10^{-12}}{0.18} = -\ 0.7J$

(b) W= $U_2 - U_1 = 0 - U = 0 - (-0.7) = 0.7 J$

Potential Energy in an External Field:

Potential energy of a single charge:



Let any source at very large distance produced an electric field $\vec{E}$ in the surrounding region. Let we bring a charge 'q' from infinity to a point in the field region where infinity to a point in the field region where potential due to source is V(r), then the work done in bringing charge 'q' from infinity to point P in field $qV(r)$.

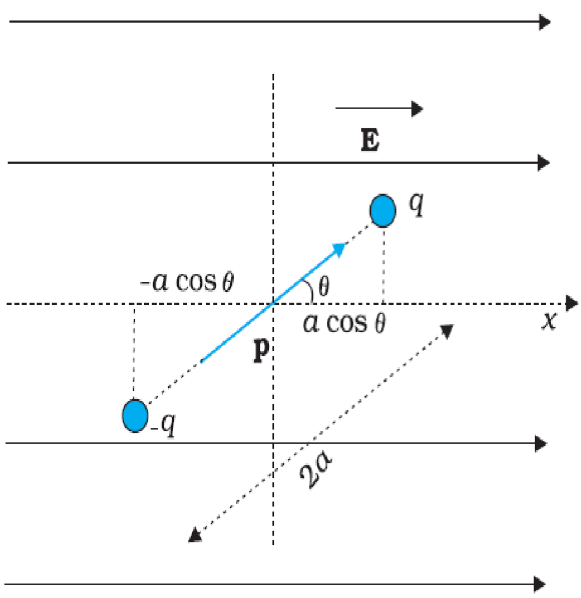
$$W = q V(r)$$

Potential energy of a dipole in a uniform external field

Consider a dipole with charges $q_1 = +q$ and $q_2 = -q$ placed in a uniform electric field E, as shown in Fig.

You will recall that in a uniform electric field, the dipole experiences no net force; but experiences a torque given by: τp × E which will tend to rotate it (unless p is parallel or anti parallel to E). Suppose an external torque $\tau(\tau_{ext})$ is applied in such a manner that it just neutralizes this torque and rotates it in the plane of paper from angle θ_0 to angle θ_1 at an infinitesimal angular speed and *without angular acceleration.*

The amount of work done by the external torque will be given by:



Potential energy of a dipole in a uniform external field.

$$W = \int_{\theta_0}^{\theta_1} \tau_{ext}(\theta)d\theta = \int_{\theta_0}^{\theta_1} pEsin\theta d\theta = pE(cos\theta_0 - cos\theta_1)$$

This work is stored as the potential energy of the system.

We can then associate potential energy U (θ) with an inclination θ of the dipole. Similar to other potential energies, there is a freedom in choosing the angle where the potential energy U is taken to be zero.

We can write:

$$U(\theta) = pE\left(cos\frac{\pi}{2} - cos\theta\right) = -pEcos\theta = -pE$$

Example:

A molecule of a substance has a permanent electric dipole moment of magnitude 10^{-29} C m. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude 10^6 V m^{-1}.

The direction of the field is suddenly changed by an angle of 60º. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarization of the sample

Solution:

Here, dipole moment of each molecules = 10^{-29} cm

As 1 mole of the substance contains 6×10^{23} molecules; then total dipole moment of all the molecules,

p = $6\times 10^{23}\times 10^{-29}$ C m= 6×10^{-6} C m

Initial potential energy, $U_i = -pE\cos\theta = -6\times 10^{-6}\times 10^6 cos\,0^0 = -6J$

Final potential energy (when, $\theta = 0^0$).

$$U_f = -6\times 10^{-6}\times 10^6 cos\,60^0 = -3J$$

Change in potential energy= -3 J- (-6 J) = 3 J

So, there is loss in potential energy.

This must be the energy released by the substance in the form of heat in aligning its dipoles.

Summary

- Electrostatic force is a conservative force. Work done by an external force (equal and opposite to the electrostatic force) in bringing a charge q from a point R to a point P is $V_P - V_R$, which is the difference in potential energy of charge q between the final and initial points.
- Potential at a point is the work done per unit charge (by an external agency) in bringing a charge from infinity to that point. Potential at a point is arbitrary to within an additive constant, since it is the potential difference between two points which is physically significant. If potential at infinity is chosen to be zero; potential at a point with position vector r due to a point charge Q placed at the origin is given is given by:

  $$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

- The electrostatic potential at a point with position vector r due to a point dipole of dipole moment p placed at the origin is:

  $$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{p\hat{r}}{r^2}$$

- The result is true also for a dipole (with charges $-q$ and q separated by: $2a$ for $r >> a$
- For a charge configuration $q_1, q_2, q_3, q_4, ...q_n$ with position vectors r_1, r_2 , ... r_n, the potential at a point P is given by the superposition principle:

  $$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + ... + \frac{q_n}{r_{nP}}\right)$$

- An equipotential surface is a surface over which potential has a constant value. For a point charge, concentric spheres centered at a location of the charge are equipotential surfaces. The electric field E at a point is perpendicular to the equipotential surface through the point. E is in the direction of the steepest decrease of potential.
- Potential energy stored in a system of charges is the work done (by an external agency) in assembling the charges at their locations. Potential energy of two charges q_1, q_2 at r_1, r_2 is given by

  $$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

  Where, r_{12} is distance between q_1 and q_2
- The potential energy of a charge q in an external potential $V(r)$ is $qV(r)$.

  The potential energy of a dipole moment p in a uniform electric field E is –P.E.